Optimal Neumann Boundary Control for a Freezing Process with Phase Change

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Introduction

The process of freezing fish to a block in a vertical plate freezer (see Figure 1) is studied.

Liquid ammonia (NH_3) at a minimum of 235 K is used as the cooling medium.

A pump forces the ammonia through the plate freezer, in which it partly vaporizes due to the heat taken off the fish block.

The amount of heat added to the ammonia is removed in a compression/condensation/throttling - process.

Thus the freezing process consists of 2 loops (see Figure 2):

- An inner *transfer loop*, where heat is transferred from the fish block to the ammonia
- An outer *regeneration loop*, where the added heat is removed from the ammonia

In practice, the freezing of fish in vertical plate freezers is done by using experience and rules of thumb to estimate the time until the interior is frozen down to at least -18 °C.

Figure 1: Vertical plate freezer

In this study, we aim to contribute to a more energy efficient way of freezing fish in vertical plate freezers by applying mathematical modeling and optimization tools.

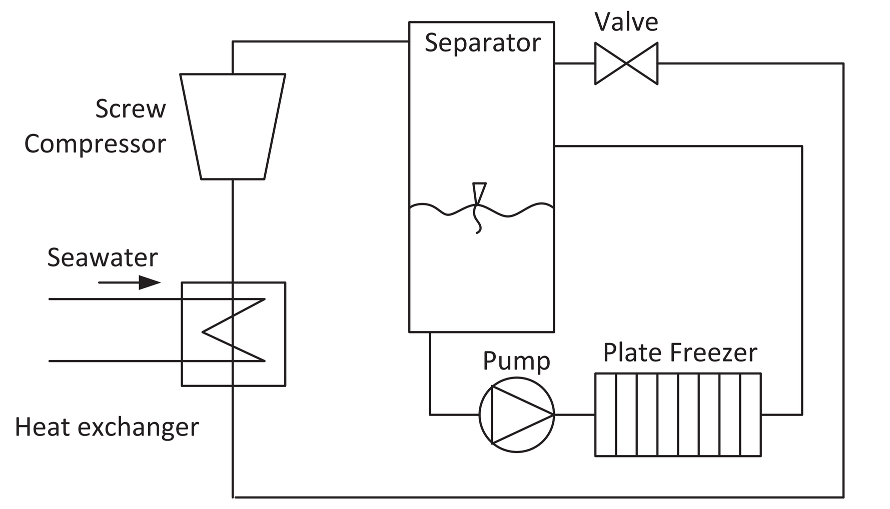

Figure 2: The cycle process with inner and outer loop

Model

The freezing process, that is the dynamics of the temperature T, is modeled by the heat equation, a nonlinear parabolic partial differential equation (PDE) (see [4] for an overview). For simplicity, and without loss of generality, only two spatial dimensions, x and y, are considered:

$$\rho(T)c(T)\frac{\partial}{\partial t}T(t,x,y) = \frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial}{\partial x}T(t,x,y)\right] + \frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial}{\partial y}T(t,x,y)\right]$$

with $\displaystyle \frac{\partial}{\partial x,y}\lambda(T) = \frac{\partial\lambda(T)}{\partial T}\frac{\partial T(t,x,y)}{\partial x,y} = \lambda_T(T)\frac{\partial T(t,x,y)}{\partial x,y}$

$$\Rightarrow \frac{\partial T(t,x,y)}{\partial t} = k_T(T)\left[\left(\frac{\partial T(t,x,y)}{\partial x}\right)^2 + \left(\frac{\partial T(t,x,y)}{\partial y}\right)^2\right] + k(T)\left[\frac{\partial^2 T(t,x,y)}{\partial x^2} + \frac{\partial^2 T(t,x,y)}{\partial y^2}\right]$$

with $\displaystyle k_T(T) = \frac{\lambda_T(T)}{\rho(T)c(T)}$ and $\displaystyle k(T) = \frac{\lambda(T)}{\rho(T)c(T)}$,

where $\rho(T)$ denotes density, $c(T)$ specific heat capacity at constant pressure and $\lambda(T)$ thermal conductivity.

Two basic phenomena have to be included in the model of the system, namely

- Latent heat of fusion which is modeled by adapting the parameters $k_T(T)$ and $k(T)$ with the *apparent heat capacity method* (see [1] and [3])
- Basic laws of thermodynamics (especially the zeroth law) which is modeled by a function that turns off heat exchange, when the temperature of the ammonia equals the temperature of the boundary layer of the fish block

To solve the problem numerically, discretization is necessary. Both, center and forward difference approaches are chosen, where n discretization steps in x-direction and m discretization steps in y-direction result in $m \times n$ ODEs (see Figure 3).

Neumann boundary conditions define heat flow through the boundaries:

- In x-direction: Optimization variables / inputs u_i defining heat exchange with the ammonia
- In y-direction: Heat exchange with air (top, $y = 0$) and perfect isolation (bottom, $y = H$)

For simplicity only one input is defined as optimization variable. Its capacity is reduced for each cell defined in y-direction, meaning that there is full capacity at the top cell and about 64% capacity at the bottom cell.

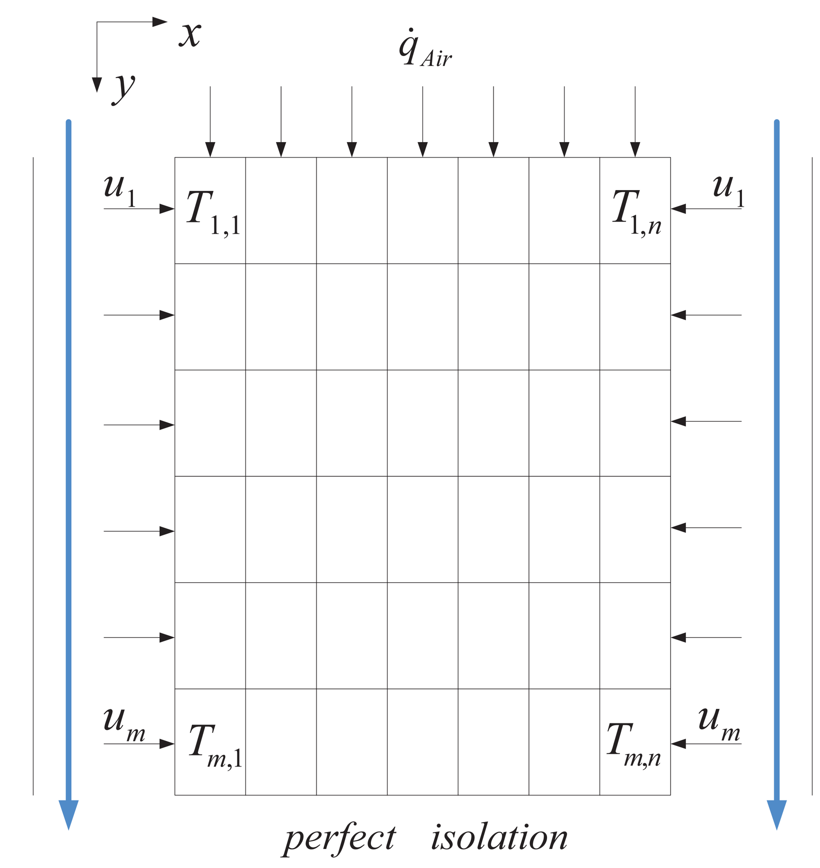

Figure 3: Discretization scheme

Results

Simulations were accomplished with ACADO for MATLAB developed by M. Diehl and co-workers, see e.g. [2].

The following settings were used for the simulations:

- $n = 9$, $m = 5$ and simulation parameters as used in [1]
- $\min\limits_{u_1} \int_0^{\tau} (T_{i,j} - T_{ref,i,j})^T Q (T_{i,j} - T_{ref,i,j}) + R(u_1 - u_{1,ref})^2 dt$ is the cost function where Q is diagonal and R is a scalar
- The function that turns off heat exchange is defined as $f(T) = \frac{1}{\pi}\left[\left[\arctan\left(\left(\frac{T}{T_{Ammonia}} - 1\right) 50000\pi\right)\right] + \frac{\pi}{2}\right]$
- The heat exchange with air is constant $\dot{q}_{Air} = -0.01 = k_{Air}\, A\, \frac{T_{Boundary} - T_{Air}}{\Delta y}$ and the heat exchange with the bottom wall is $\dot{q}_{Bottom} = 0$
- The piecewise constant optimization variable u_1 is constrained by upper and lower bounds
- Single shooting with 300 equidistant time instances

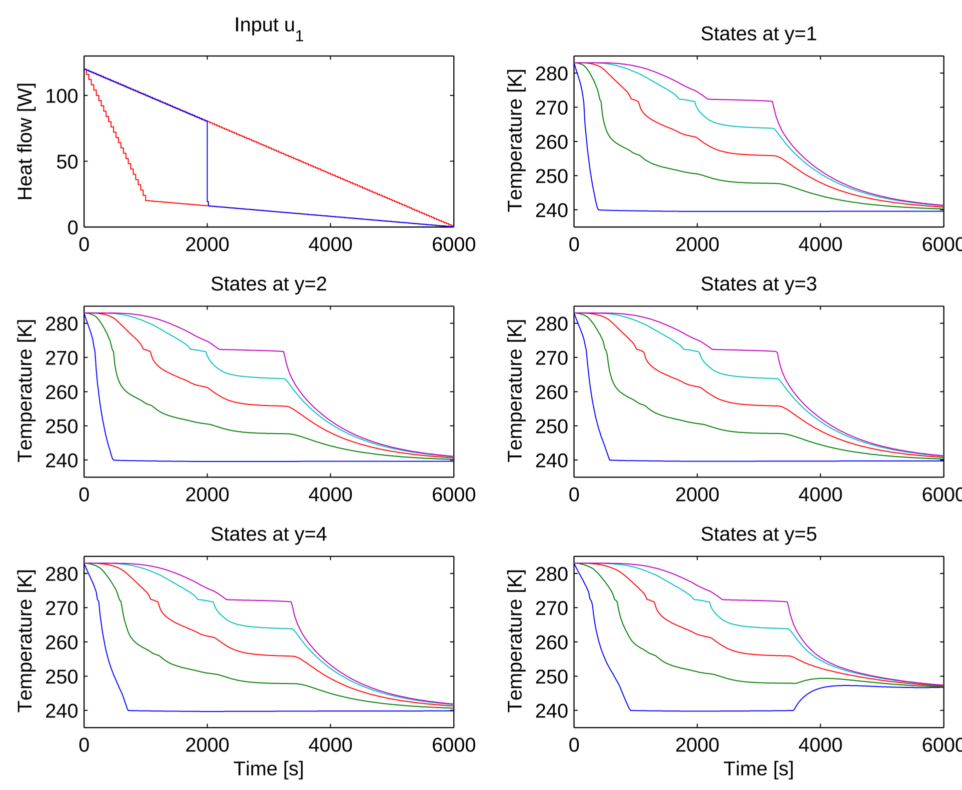

Figure 4: Input u_1 without weighted reference ($R = 0$) and states at different positions y and x

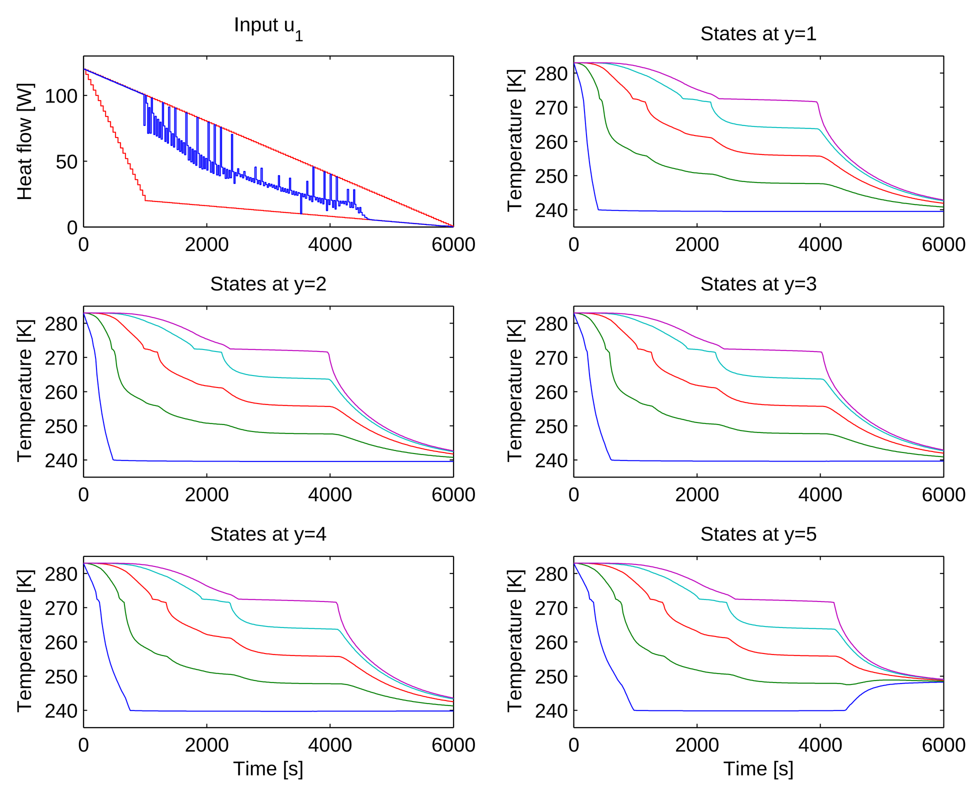

Figure 5: Input u_1 with weighted reference ($R \neq 0$, $u_{1,ref} = 0$) and states at different positions y and x

Discussion

The above presented results were achieved by several simplifications:

- The initial condition is equally distributed ($T_{init} = 283$ K)
- Consideration of two spatial dimensions without loss of generality
- The plate freezer is perfectly isolated at the bottom
- The heat exchange with air is constant and not depending on temperature difference
- The values of the simulation parameters are approximations of the real values
- The fish in between the freezing plates is considered as a homogenous mass without any entrapped air
- Only one input is defined as optimization variable

Future work:

- Definition of a unique input for any discretization cell in y-direction
- Constraints on the gradients of the inputs $\Rightarrow$ new optimization variables
- Development of a closed loop description with temperature feedback to get a more accurate calculation of the heat exchange
- Validation of the simulation results by measurements
- Definition of a 3-dimensional description of the process
- Consideration of more than one plate freezer cell and many plate freezers in parallel
- Integration of terminal constraints

References

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[3] M. Muhieddine, É. Canot, and R. March. Various approaches for solving problems in heat conduction with phase change. *International Journal of Finite Volume Method*, 6(1), 2008.

[4] Q. T. Pham. Modelling heat and mass transfer in frozen foods: a review. *International Journal of Refrigeration*, 29:876–888, 2006.